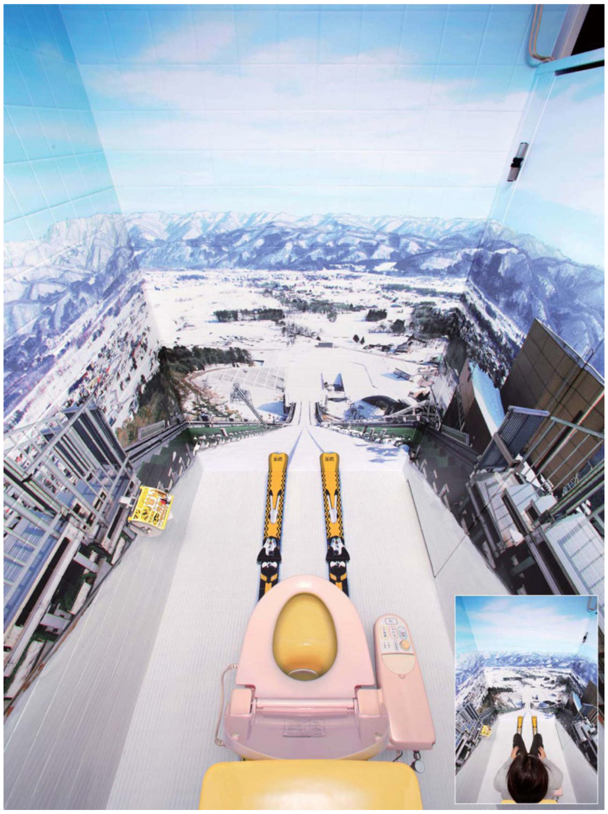
Two Dimensional Dynamics



So you say you'd like more hands-on learning?

Physics courses love to talk about objects sliding up and down frictionless slopes.

This is the view from the top of a ski jump. As the skiers move down the inclined plane (the slope), they will be moving both vertically and horizontally and will experience forces in both dimensions.

For some of us, just sitting at the top can be a traumatic experience. In my case, I'd rather work the problem on paper rather than get hands on experience here.

*D**ynamics** in two dimensions uses the same ideas that were covered in one dimension. There are a few tips and tricks that will make the work go easier though.*

In the end, you need to be able to break vectors that are not aligned with your coordinate system into their components and sometimes you need to take the components and use them to create two dimensional vectors.

Net Force in Two Dimensions

Pulling a pig in two different directions. What is the net force? Given to a class just learning Basic Right Triangle Trigonometry.

Free Body Diagrams

A tightrope walker is balancing at a point where the rope in front of him angles up at 5.0 deg above the horizontal and the rope behind him angles upward at 15 deg above the horizontal.

Draw a free body diagram of the spot on the tightrope where the acrobat is standing.

A safe sitting on the floor is going to be pushed to the right and downward.

Draw the free body diagram if the floor is frictionless.

Draw a free body diagram of the same situation if the floor is not frictionless.

A skier is gliding down a hill as shown here.

Draw a free body diagram of the skier on the hill if there is no friction.

Draw a free body diagram of the same skier with friction between the skier and the slope.

An airplane is gliding forward at a constant altitude. The force that keeps it up (pushes it upward) is called lift. The lift comes from the direction of the wings, it is perpendicular to the wings.

Draw a free body diagram of the forces on this airplane if there is no air friction.

Draw a free body diagram of the same airplane (gliding forward) with air friction slowing it down.

A second airplane banks its wings to the right in order to turn. The lift force is still perpendicular to the wings.

Draw a free body diagram of the forces acting on the airplane as it banks right (assume no air friction to slow it down).

Newton in Two Dimensions

Applying Newton's Laws in two dimensions starts and ends with ***breaking the problem up into separate but related one dimensional problems***.

You are going to

- Draw the free body diagram,
- Draw an appropriate 2D coordinate system,
- Use the components of the force vectors that are parallel to the coordinate axes

to solve the problems.

It's just that simple. Students who struggle with them usually have not drawn a correct free body diagram or they have no coordinate system accompanying it, or they will not break the vector into components and work two problems instead of one.

Free Body Diagrams

Solving two dimensional problems using Newton's laws starts with the free body diagram. The idea is the same as it was in one dimension.

Isolate (free) the object (the body) from its surroundings and draw only the forces that are acting on it.

Make sure you get the directions correct.

Try to draw the diagram with some sense of the relative sizes of the vectors.

If you are dealing with an object that is not accelerating in any direction, then all of the vectors should add to produce zero. That means if you started with one of the vectors and added them head to tail, you would eventually end up right back where you started (for zero net force).

The 2D Coordinate System

Once you have a correct free body diagram, it is time to pick a workable coordinate system. You should have been doing this in one dimensional dynamics anyway.

The selection of a usable coordinate system is often the difference between solving the problems and being lost. If you move on in physics (past high school) that might even mean using coordinates that are not Cartesian (x and y). For us, we will stick with the cartesian coordinate system for this unit.

Since we are going to be sticking with the Cartesian coordinate system. You need to decide where to put the origin and how to orient your axes (should you rotate them or not). Rotation of the axes is discussed in more detail below, but for now, you should make sure to align your coordinate system so that the direction of the motion that you are investigating is parallel to either the x or the y axis. If it is not, then you will have to rotate the coordinate system until one axis is aligned with (parallel to) the direction of motion.

On the other hand, if the motion is going to be parallel to your x or y axis, do not rotate them!

Components of Force Vectors and Newton's 2nd Law

After you have drawn a correct free body diagram and have drawn the coordinate axes, you will need to look for all of the vectors on your free body diagram that are not parallel to one of the axes and break it up into its components along the axes.

Breaking up the vectors into components aligned with the axes creates two one dimensional problems. You already have the tools and skills to solve one dimensional problems; one in the x-direction and one in the y-direction.

Finally, using Newton's second law, add all the forces along each axis to determine how the mass in your free body diagram will accelerate along that axis.

A boulder is being pushed up a hill as shown here. There is friction between the boulder and the hill.

Draw a free body diagram of the forces acting on the boulder as it is being pushed.

Draw a free body diagram of the man pushing the boulder AS he is pushing it.

Balanced Forces in 2 Dimensions

Tension in 3 Ropes (Strings). This lecture helps you to setup the problem. You still have to figure out the algebra to finish it!

Coordinates and Components

Back to the tight-rope walker using the free body diagram picture.

Draw another free body diagram with a coordinate system. Break up the force vectors that are not aligned with the axes into components in the x and y directions.

What the sum of all three forces acting on the point below the man's feet?

If the man is 93 kg, what is the tension in the two parts of the rope?

We are pushing the safe across the floor again. Look at the free body diagram here.

Draw a coordinate system that has one of the axes aligned with the direction of motion and then determine which force vectors need to be broken into components to match the coordinate system.

What minimum pushing force at a 20 degree downward angle is required to just start the safe sliding. The safe is 150 kg and the coeffecient of static friction is 0.65.

Use the free body diagram of the airplane that is banking to it's right.

Draw a coordinate system that will have the x-axis aligned with the direction of acceleration (assume that the plane stays at a constant altitude).

Draw the free body diagram of this same airplane as it turns to the right and break up any force vectors that are not aligned with the axes.

If the airplane is a Boeing 737 (mass of 44,700kg), what angle should the airplane tilt at in order to accelerate to its right at 5.5 m/s?

Use the free body diagram of the boulder that is being pushed up the hill.

Draw a 2-dimensional cartesian coordinate system that will allow you to explain the motion of the boulder.

Sliding Downhill

Setting up the problem with and without friction.

Slopes and Inclines

A skier is sliding frictionlessly down a slope as shown here.

Draw a free body diagram for the skier.

Draw a coordinate system on the free body diagram. That means one of the axes is parallel to the direction of the motion.

What is the acceleration of the skier down the hill? (No numbers. Just letters.)

The skier is 75kg. The slope is 38 degrees. If the skier started from rest, what is the speed of the skier after 1.3 seconds?

This problem has a block resting on an inclined surface as shown. There is friction between the block and the surface. The slope is increased until the block starts to slide.

Draw the picture with rotated axes.

Draw a free body diagram using the rotated axes.

Break up the components of the blocks weight on your free body diagram so that the forces are all aligned with the new coordinate axes.

What is the coeffecient of static friction?

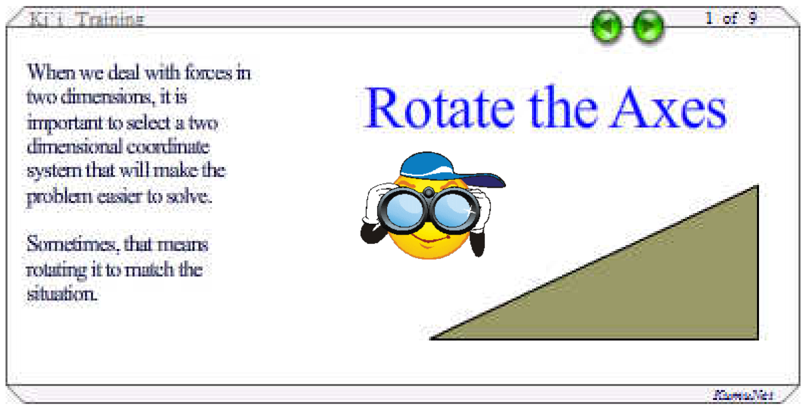
Now that the block is accelerating downhill, how could we change the slope in order to find the coeffecient of kinetic friction?

Rotation of the Axes

Whenever the motion of the object is going to move up or down a slope in 2 dimensions, we are going to rotate the axes so that the x-axis is parallel to the slope In other words, the x-axis will now be parallel to the direction that we anticipate the motion to be in. By doing this, we can still describe our motion in a single dimension even though the original problem used two dimensions.

Slopes, Inclines, Hills

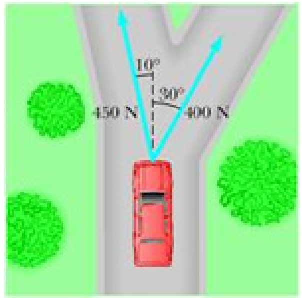
The following training animation describes the process of rotation and how the new axes and vector components are related to the angle of the slope.



Now it's just a matter of working problems. Lots of them! Let's get started.

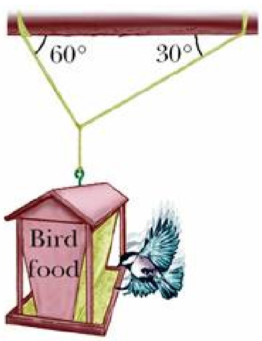
Problems

1. Two forces are applied to a car in an effort to move it, as shown in the figure. (a) What is the resultant of these two forces? (b) If the car has a mass of 3,000 kg, what acceleration does it have? Ignore friction.

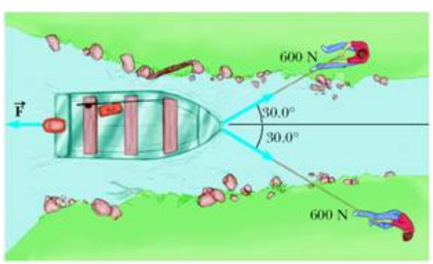


2. The force exerted by the wind on the sails of a sailboat is 390 N to the north. The water exerts a force of 180 N east. If the boat (including its crew) has a mass of 270 kg, what are the magnitude and direction of its acceleration?

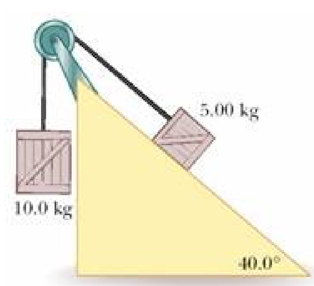
3. A 150-N bird feeder is supported by three cables as shown in the figure. Find the tension in each cable.



4. Two people are pulling a boat through the water as in the figure. Each exerts a force of 600 N directed at a 30.0° angle relative to the forward motion of the boat. If the boat moves with constant velocity, find the resistive force exerted by the water on the boat.

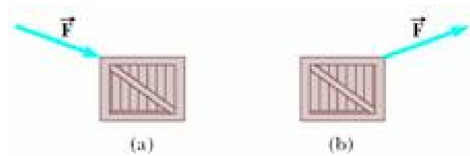


5. Two packing crates of masses 10.0 kg and 5.00 kg are connected by a light string that passes over a frictionless pulley as in the figure. The 5.00-kg crate lies on a smooth (frictionless) incline of angle 40.0°. Find the acceleration of the 5.00-kg crate and the tension in the string.

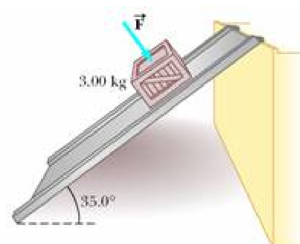


6. A 40.0-kg wagon is towed up a hill inclined at 18.5° with respect to the horizontal. The tow rope is parallel to the incline and has a tension of 140 N. Assume that the wagon starts from rest at the bottom of the hill, and neglect friction. How fast is the wagon going after moving 80.0 m up the hill?

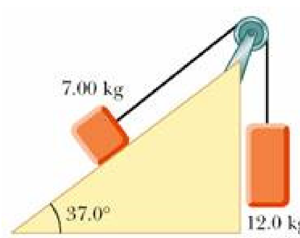
7. A 1 000-N crate is being pushed across a level floor at a constant speed by a force of 300 N at an angle of 20.0° below the horizontal, as shown in the figure. (a) What is the coefficient of kinetic friction between the crate and the floor? (b) If the 300-N force is instead pulling the block at an angle of 20.0° above the horizontal, as shown in the figure, what will be the acceleration of the crate? Assume that the coefficient of friction is the same as that found in (a).



8. The coefficient of static friction between the 3.00-kg crate and the 35.0° incline of the figure is 0.300. What minimum force must be applied to the crate perpendicular to the incline to prevent the crate from sliding down the incline?



9. Find the acceleration reached by each of the two objects shown in the figure if the coefficient of kinetic friction between the 7.00-kg object and the plane is 0.250.



10. A 4.00-kg block is pushed along the ceiling with a constant applied force of 85.0 N that acts at an angle of 55.0° with the horizontal, as in the

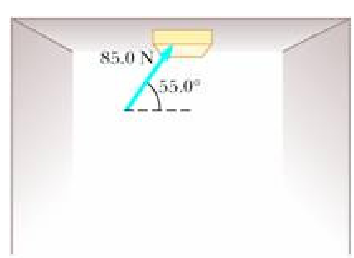


figure. The block accelerates to the right at 6.00 m/s2. Determine the coefficient of kinetic friction between block and ceiling.

11. A 3.0-kg object hangs at one end of a rope that is attached to a support on a railroad car. When the car accelerates to the right, the rope makes an angle of 4.0° with the vertical, as shown in the figure. Find the acceleration of the car.